



# Research Journal of Pharmaceutical, Biological and Chemical Sciences

## Considering Elasticity Uncertainty in Customer's Response to Time-Of-Use Options in Electricity Markets

Tahereh Entezari Harsini<sup>1\*</sup>, Azam Entezari Harsini<sup>2</sup>, and Iraj Hemmati<sup>3</sup>.

<sup>1,2,3</sup> Department of Electrical Engineering, College of Engineering, Islamshahr Branch, Islamic Azad University, Tehran, Iran.

### ABSTRACT

This paper investigates the impact of demand side model in demand response programs (DRPs). A power modeling is assumed for time of use (TOU) demand response programs. The nonlinear behaviors of elastic loads as well as uncertainty in forecasted elasticity are incorporated in the modeling. These considerations help to increase the flexibility and viability of the presented model. The methodology is demonstrated through case study simulated on Iranian power system. Obtained results on the proposed system show the great impact of running TOU programs using proposed power model on load profile of the peak day of the Iranian power system.

**Keywords:** Demand Response programs, Elasticity, Time-of-use programs.

*\*Corresponding author*

## INTRODUCTION

Time-based pricing is a pricing strategy where the provider of a service or supplier of a commodity may vary the price depending on the time-of-day when the service is provided or the commodity is delivered. The rational background of time-based pricing is expected or observed change of the supply and demand balance during time. Time-based pricing includes fixed time-of use rates for electricity and public transport, dynamic pricing reflecting current supply-demand situation or differentiated offers for delivery of a commodity depending on the date of delivery (futures contract). Most often time-based pricing refers to a specific practice of a supplier. The mentioned concept has been depicted in figure 1.

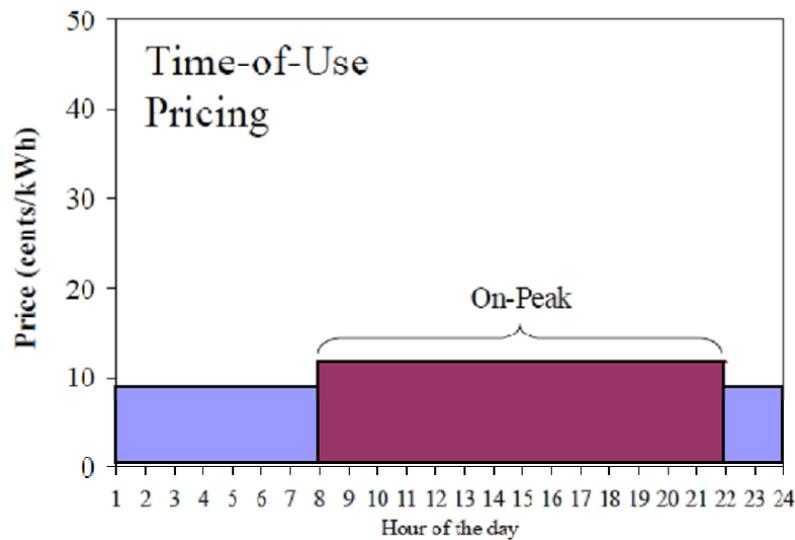


Figure 1: Hourly pricing of electricity (time of use pricing)

Time-of-use pricing (TOU pricing), whereby electricity prices are set for a specific time period on an advance or forward basis, typically not changing more often than twice a year. Prices paid for energy consumed during these periods are preestablished and known to consumers in advance, allowing them to vary their usage in response to such prices and manage their energy costs by shifting usage to a lower cost period or reducing their consumption overall;

Time of use programs (TOU) are the most prevalent priced-based programs. Most customers are exposed to some form of TOU rates, if only with rates that vary by six-month seasons. For instance, a summer-peaking utility may charge a higher rate for the energy use part of a bill than for the same amount of electricity consumed during the off-peak six months. This is a seasonal (time-varying) rate. More sensitive time-of-use rates establish two or more daily periods that reflects hours when the system load is higher (peak) or lower (off-peak), and charge a higher rate during peak hours. The definition of TOU periods differs widely among utilities, based on the timing of their peak system demands over the day, week, or year. TOU rates sometimes have only two prices, for peak and off-peak periods, while other tariffs include a shoulder period or partial-peak rate [1, 2].

Considerable researches have been done to introduce and extend linear economic modeling of DRPs [3-6]. The previous models have two main defects. First the models are linear that could say customer behavior is not linear and could be more complicated than linear behavior in many real situations, It could be mentioned that based on nonlinear behavior of real demand, those models don't be able to simulate demand responses accurately and second is about demand elasticity which has modeled as a fixed value while there is some extent uncertainty in forecasted elasticity of demand. These problem have been solved in this paper because they could help the model becomes more realistic.

In this paper, a power model to describe price dependent loads is developed such that the characteristics of TOU programs can be imitated. The remaining parts of the paper are organized as following: the definition of elasticity is reviewed in section 2. Section 3 is about elasticity uncertainty. Power modeling of DR based on the concept of price elasticity of demand is developed in section 4. Section 5 is devoted to simulation results where the impact of TOU programs via proposed exponential model on load profile of the peak day of the Iranian power system in 2007 is investigated. Finally, the paper is concluded in section 5.

**Elasticity definition**

Generally, electricity consumption like most other commodities, to some extent, is price sensitive. This means when the total rate of electricity decreases, the consumers will have more incentives to increase the demand. This concept is shown in figure 2, as the demand curve.

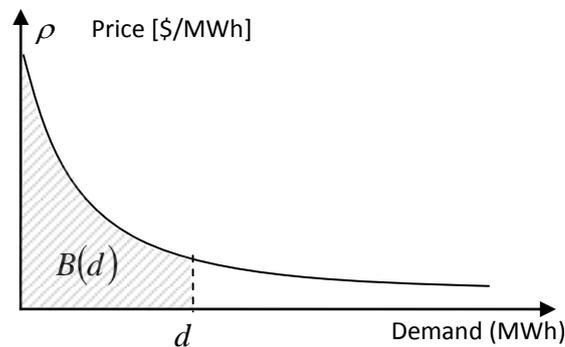


Figure 2: Demand curve

Hachured area in fact shows the customer marginal benefit from the use of  $d$  MWh of electrical energy. This is represented mathematically by:

$$B(d) = \int_0^d \rho(d) \cdot \partial d \tag{1}$$

Based on economics theory, the demand-price elasticity can be defined as follows:

$$e = \frac{\Delta d / d}{\Delta \rho / \rho^0} \tag{2}$$

For time varying loads, for which the electricity consumptions vary during different periods, cross-time elasticity should also be considered. Cross-time elasticity, which is represented by cross-time coefficients, relates the effect of price change at one point in time to consumptions at other time periods. The self-elasticity coefficient,  $e_{tt}$ , (with negative value), which shows the effect of price change in time period  $t$  on load of the same time period and the cross-elasticity coefficient,  $e_{t\hat{t}}$ , (with positive value) which relates relative changes in consumption during time period  $t$  to the price relative changes during time period  $\hat{t}$  are defined by following relations:

$$e_{tt} = \frac{\partial d_t / d_t}{\partial \rho_t / \rho_t} \tag{3}$$

$$e_{t\hat{t}} = \frac{\partial d_t / d_t}{\partial \rho_{\hat{t}} / \rho_{\hat{t}}} \tag{4}$$

**Elasticity uncertainty**

The expression for the probability density function of a normal distribution is always written as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x - \mu)^2}{2\sigma^2}\right] \tag{5}$$

Where  $\mu$  is mean value and  $\sigma$  is standard deviation.

Typical normal density function curves are shown in Figure 3 for a given value of  $\mu$  and three values of  $\sigma$ . Similarly since the value of  $\sigma$  determines the amount of spread or dispersion and therefore the shape of the curve, it is referred to as the scale parameter.

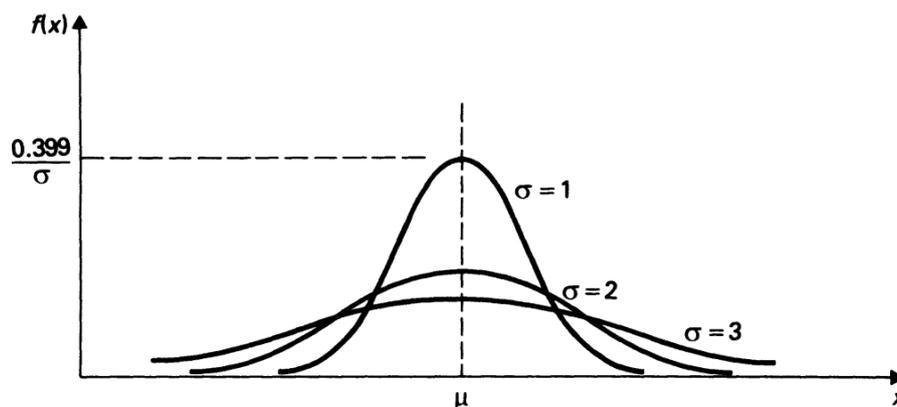


Figure 3: Normal density functions for three values of  $\sigma$  .

It is extremely difficult to obtain sufficient historical data to determine the distribution describing the elasticity coefficient uncertainty. Published data, however, has suggested that the uncertainty can be reasonably described by a normal distribution. The distribution mean is the forecasted elasticity coefficient. The distribution can be divided into a discrete number of class intervals. The elasticity representing the class interval mid-point is assigned the designated probability for that class interval. This is shown in Figure 4, where the distribution is divided into seven steps. A similar approach can be used to represent a non symmetrical distribution if required. It has been found that there is little difference in the end result between representing the distribution of elasticity coefficient uncertainty by seven steps or forty-nine steps. Here, we consider eleven intervals because our simulation showed that considering higher step numbers have no effect on the end result.

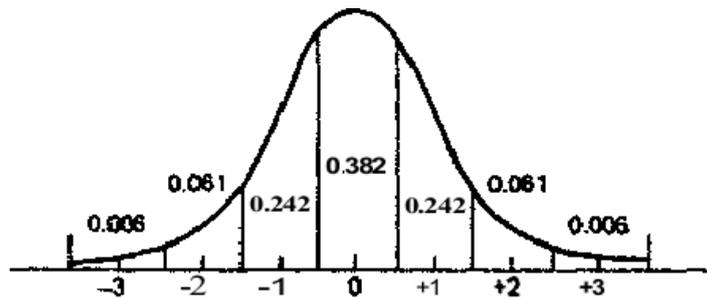


Figure 4: Seven-step approximation of the normal distribution

**Power modeling of elastic loads**

The proper offered rates can motivate the participated customers to revise their consumption pattern from the initial value  $d_t^0$  to a modified level  $d_t$  in period t.

$$\Delta d_t = d_t - d_t^0 \tag{6}$$

It is reasonable to assume that customers will always choose a level of demand  $d_t$  to maximize their total benefits which are difference between incomes from consuming electricity and incurred costs; i.e. to maximize the cost function given below:

$$B[d_t] - d_t \cdot \rho_t \tag{7}$$

The necessary condition to realize the mentioned objective is to have:

$$\frac{\partial B[d_t]}{\partial d_t} - \rho_t = 0 \tag{8}$$

Thus moving the last term to the right side of the equality,

$$\frac{\partial B[d_t]}{\partial d_t} = \rho_t \tag{9}$$

Substituting (8) to (3) and (4), a general relation based on self and cross elasticity coefficients is obtained for each time period t as follows:

$$\frac{\partial d_t}{d_t} = e_{tt} \frac{\partial \rho_t}{\rho_t} \tag{10}$$

By assuming constant elasticity for NT-hours period,  $e_{tt} = \text{Constant}$  for  $t, \acute{t} \in \text{NT}$  integration of each term, we obtain the following relationship.

$$\int_{d_t^0}^{d_t} \frac{\partial d_t}{d_t} = \sum_{\acute{t}=1}^{NT} \left\{ e_{tt} \left[ \int_{\rho_t^0}^{\rho_t} \frac{\partial \rho_t}{\rho_t} \right] \right\} \tag{11}$$

Combining the customer optimum behavior that leads to (8), (9) with (10) yields the power model of elastic loads, as follows:

$$d_t = d_t^0 \prod_{\acute{t}=1}^{NT} \left( \frac{\rho_t}{\rho_t^0} \right)^{e_{tt}} \tag{12}$$

Parameter  $\eta$  is demand response potential which can be entered to model as follows:

$$d_t = d_t^0 + \eta d_t^0 \left\{ \prod_{\acute{t}=1}^{NT} \left( \frac{\rho_t}{\rho_t^0} \right)^{e_{tt}} - 1 \right\} \tag{13}$$

The larger value of  $\eta$  means the more customers' tendency to reduce or shift consumption from peak hours to the other hours.

Elasticity uncertainty based on details explained in the previous section could be considered in the equation number (13) as follow:

$$d_t = \sum_{i=1}^{ND} \left[ \text{prob}_i \left( d_t^0 + \eta d_t^0 \left( \prod_{\acute{t}=1}^{NT} \left( \frac{\rho_t}{\rho_t^0} \right)^{e_{tt,i}} - 1 \right) \right) \right] \tag{14}$$

Where  $ND$  is the number of class intervals,  $\text{prob}_i$  is probability of class interval  $i^{th}$  and  $e_{tt,i}$  is mid-point of class interval  $i^{th}$ .

### Simulation results

In this section numerical study for evaluation of proposed model of EDRP programs are presented. For this purpose the peak load curve of the Iranian power grid on 28/08/2007 (annual peak load), has been used for our simulation studies [13]. Also the electricity price in Iran in 2007 was 150 Rials (Unit of Iranian currency). This load curve, shown in figure 5, divided into three different periods, namely valley period (00:00 am–9:00 am), off-peak period (9:00 am–7:00 pm) and peak period (7:00 pm–12:00 pm).

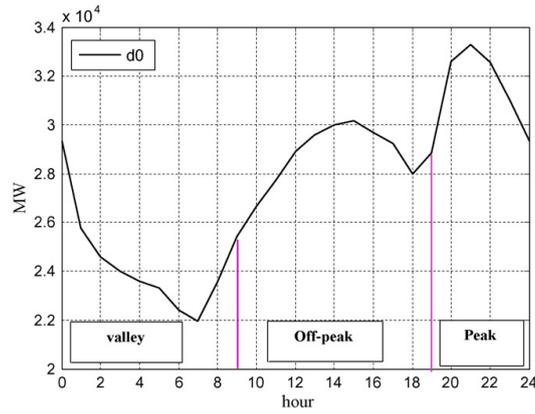


Figure 5: Initial load profile

The selected values for the self and cross elasticities have been shown in Table 1.

Table 1: Self and cross elasticities

	Low	Off-peak	Peak
Low	-0.10	0.014	0.016
Off-peak	0.014	-0.10	0.012
Peak	0.016	0.012	-0.10

Different scenarios are considered according to Table 2.

Table 2: The considered scenarios

Scenario number	TOU rates (Rials/MWh)	Demand response potential (%)
1	40, 160, 400 at valley, off peak and peak periods respectively	10%
2	40, 160, 400 at valley, off peak and peak periods respectively	20%

Figure 6 shows the percent of energy reduction in different scenarios. In both scenarios, energy reductions are negative. It means that these scenarios lead to energy consumption increasing. This effect is higher in scenario 2 than other one. Another thing that could be concluded from the figure is that increasing  $\sigma$  which demonstrates higher uncertainty, leads to more energy increasing. System operators should be cautious about uncertainty that exists in forecasted elasticity because more uncertainty makes worse energy consumption condition.

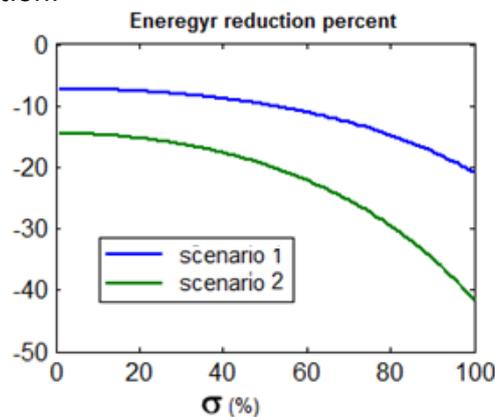


Figure 6: Energy reduction (%) in different scenarios

Figure 7 shows energy shifting (in percent) in different scenarios. Increasing demand response potential could increase shift of energy from one period to another one within daily interval. Increasing  $\sigma$  results in decreasing the mentioned effect.

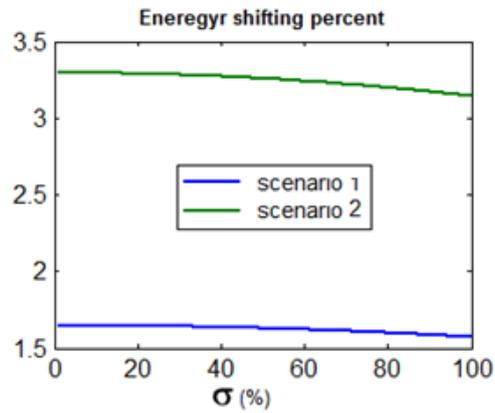


Figure 7: Energy shifting (%) in different scenarios

Figure 8 shows peak reduction (in percent) in different scenarios. This figure is the same as the former. Increasing demand response potential could increase peak reduction. Increasing  $\sigma$  has a slight decreasing effect on peak reduction.

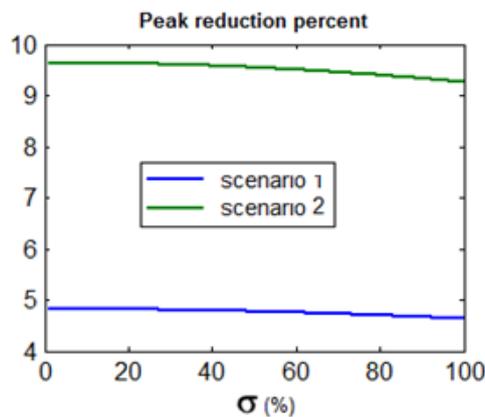


Figure 8: Peak reduction (%) in different scenarios

### CONCLUSION

In this paper, a power model of demand response program has been introduced. The main aspect of this model is consideration of uncertainty in elasticity coefficient and adding it to the model. It has been shown that this model could imitate customers' response to TOU program as prevalent DRPs. This model can help sponsor's TOU programs to simulate the behavior of customers for the purpose of improvement of load profile characteristics as well as satisfaction of customers.

### ACKNOWLEDGEMENT

The authors gratefully acknowledge the financial and other support of this research, provided by Islamic Azad University, Islamshahr Branch, Tehran, Iran.

## Nomenclature

0	Initial state index (Superscript)
$t, \hat{t}$	Time period indices (subscript)
ND	Number of class intervals of normal density function
NT	Number of hours within period of study
d	Load (MW)
$\rho$	Price (Rials/MWh)
$\Delta d$	Demand change (MW)
$\Delta \rho$	Price change ( Rials/MWh)
$B[d_t]$	Benefit of consumer at time period $t$ by consuming $d_t$
$e_{tt}$	Self elasticity
$e_{t\hat{t}}$	Cross elasticity
$e_{tt,i}$	Self elasticity in interval class $i^{th}$
$e_{t\hat{t},i}$	Cross elasticity in interval class $i^{th}$
$prob_i$	Probability of class interval $i^{th}$
$\eta$	Demand response potential (%)

## REFERENCES

- [1] FERC report. Regulatory commission survey on demand response and time based rate programs/tariffs; August 2006. <[www.ferc.gov](http://www.ferc.gov)>.
- [2] FERC report. Regulatory commission survey on demand response and time based rate programs/tariffs; 2008. <[www.ferc.gov](http://www.ferc.gov)>.
- [3] Goel L, Wu Q, Wang P. Electric Power Systems Research 2008;78:1655–1663.
- [4] Faruqui A, George S. Electricity J 2005;18(4): 53–63.
- [5] Schweppe F, Caramanis M, Tabors R, Bohn R. Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell MA, 1988.
- [6] Schweppe F, Caramanis M, Tabors R. IEEE Trans Power App Syst 1985;104(7):1644–1655.
- [7] I.R. of IRAN, Ministry of Energy, Statistical information on energy balance 2007. <<http://www.iranenergy.org.ir>>.